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DEPARTMENT OF MECHANICAL ENGINEERING AND AERONAUTICS  
SCHOOL OF ENGINEERING  
OLD DOMINION UNIVERSITY  
NORFOLK, VIRGINIA

SELF-SHADOWING OF ORBITING TRUSSES

by

Jack Mahany

and

Earl A. Thornton, Principal Investigator

Progress Report  
for the period ending May 15, 1963

Prepared for the  
National Aeronautics and Space Administration  
Langley Research Center  
Hampton, Virginia

Under  
Research Grant NAG1-257  
L. Bernard Garrett, Technical Monitor  
Space Systems Division

August 1963

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Submitted by the  
Old Dominion University Research Foundation  
P.O. Box 6369  
Norfolk, Virginia 23508



August 1983



# SELF-SHADOWING OF ORBITING TRUSSES

By

Jack Mahaney<sup>1</sup> and Earl Thornton<sup>2</sup>

## SUMMARY

- Purpose - Determine shadowing reductions on heating of orbiting trusses
- Scope - Determination of heating rates with slender member shadowing effects included
- Thermal response of shadowed member

## Analysis Approach

1. Determine shadowing
2. Heating analysis
3. Thermal analysis
4. Structural analysis

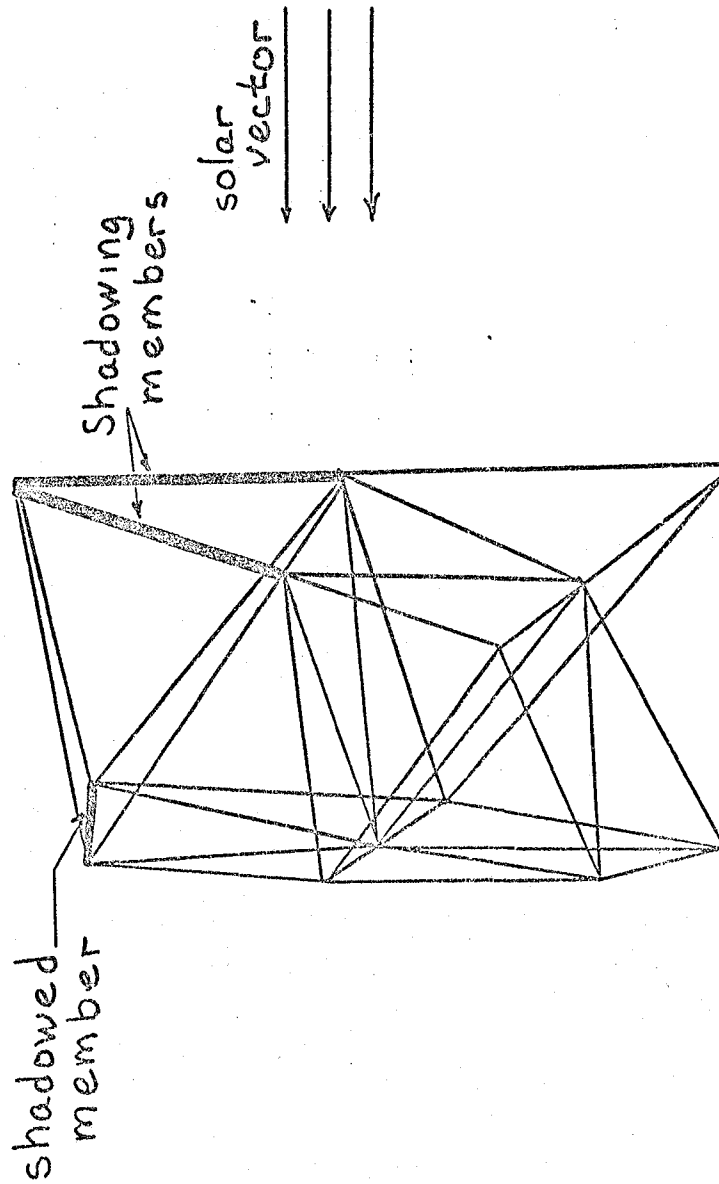
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<sup>1</sup> Graduate Assistant, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508.

<sup>2</sup> Associate Professor, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508.

# TYPICAL TRUSS SHADOWING

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APPROACH

1. Subdivide shadowed member

truss member

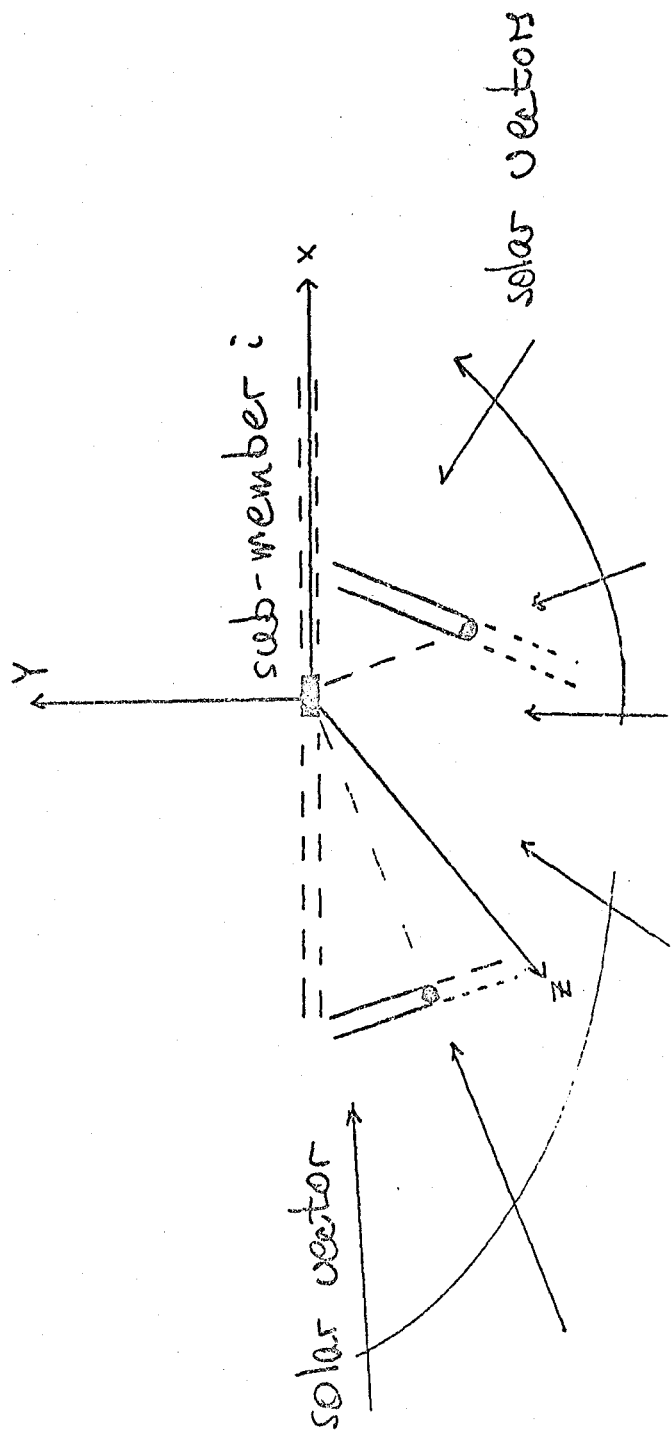


sub-member

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## APPROACH

2. Calculate centroid of sub-member  $i$ .
3. Translate nodal co-ordinates
4. Identify shadowers
5. Determine where shadowing occurs.





## APPROACH

### 6. Calculate shadow intensity (SHAD)

$$\text{SHAD} = f(a, b, s, R_s)$$

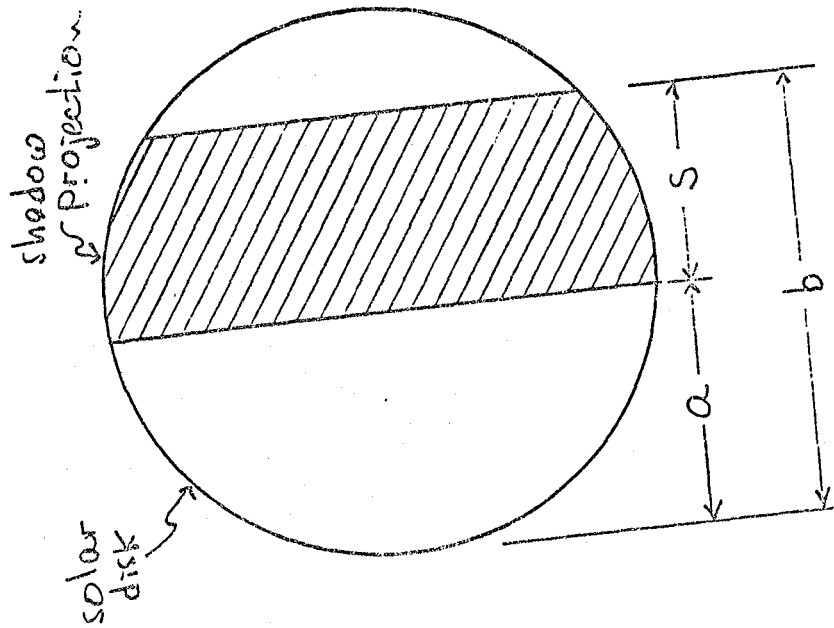
$R_s$  = Radius of Sun

SHAD is a measure of  
shadow intensity.

= 1, no shadow

= 0, full shadow

$$\text{incident solar heat} = \text{solar flux} \times \text{SHAD}$$

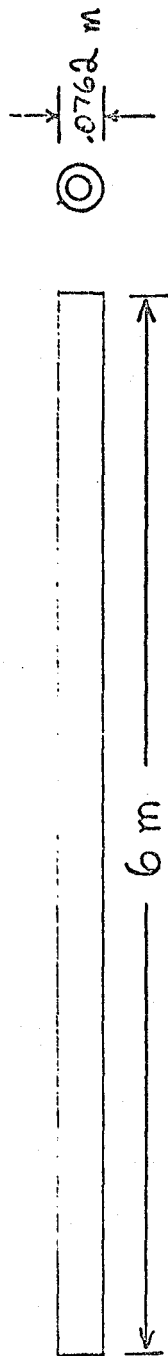


# APPROACH

1. Subdivide truss member
2. Calculate centroid of sub-member  $i$ .
3. Translate nodal co-ordinates to  $i$ -centered system
4. Identify shadowers
5. Find locations where shadowing occurs
6. Calculate shadow intensity (SHAD)

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# Typical Truss Member

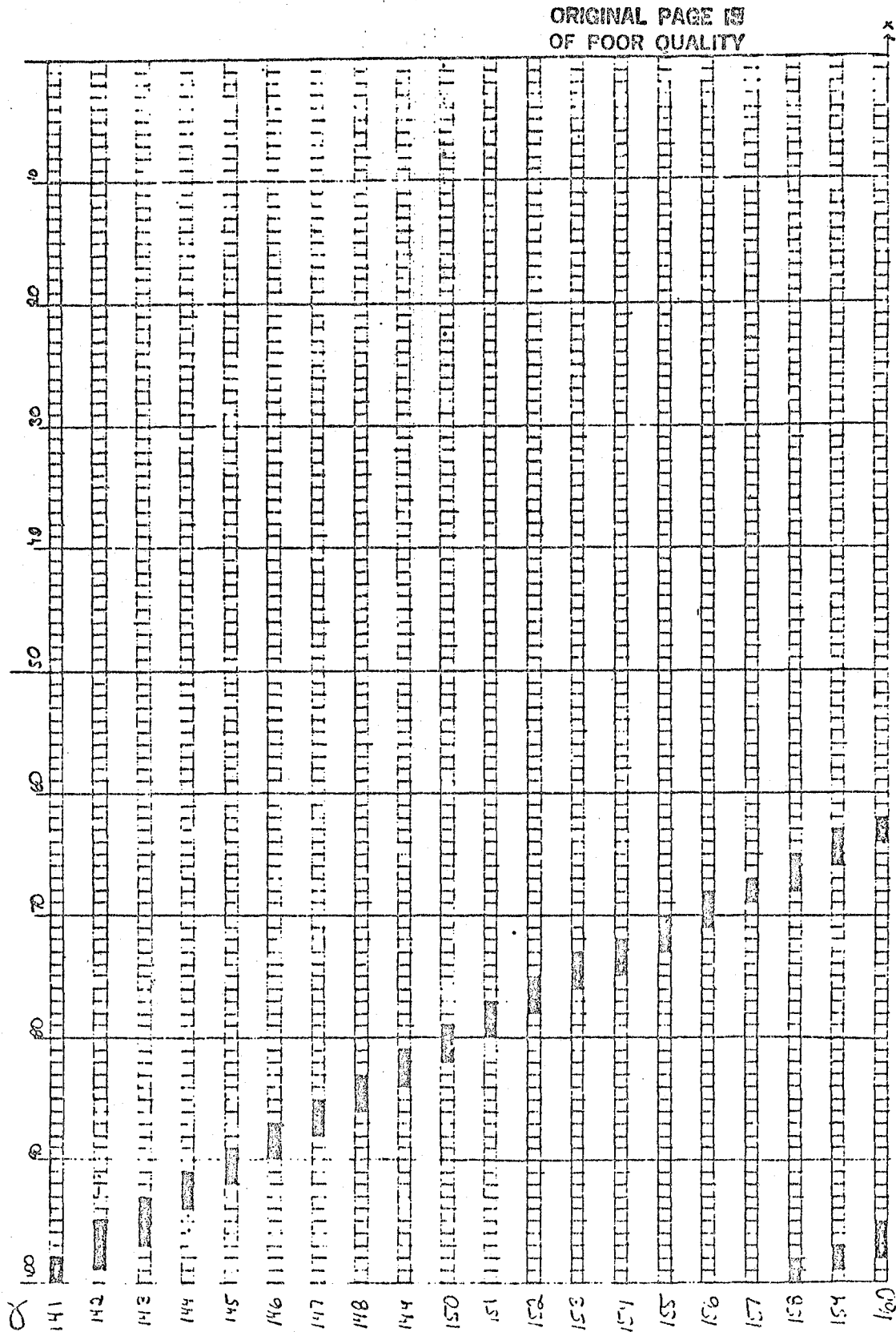


Cross-Sectional Area  $.171 \text{E}-3 \text{ m}^2$   
 Radiating Perimeter  $.2395 \text{ m}$

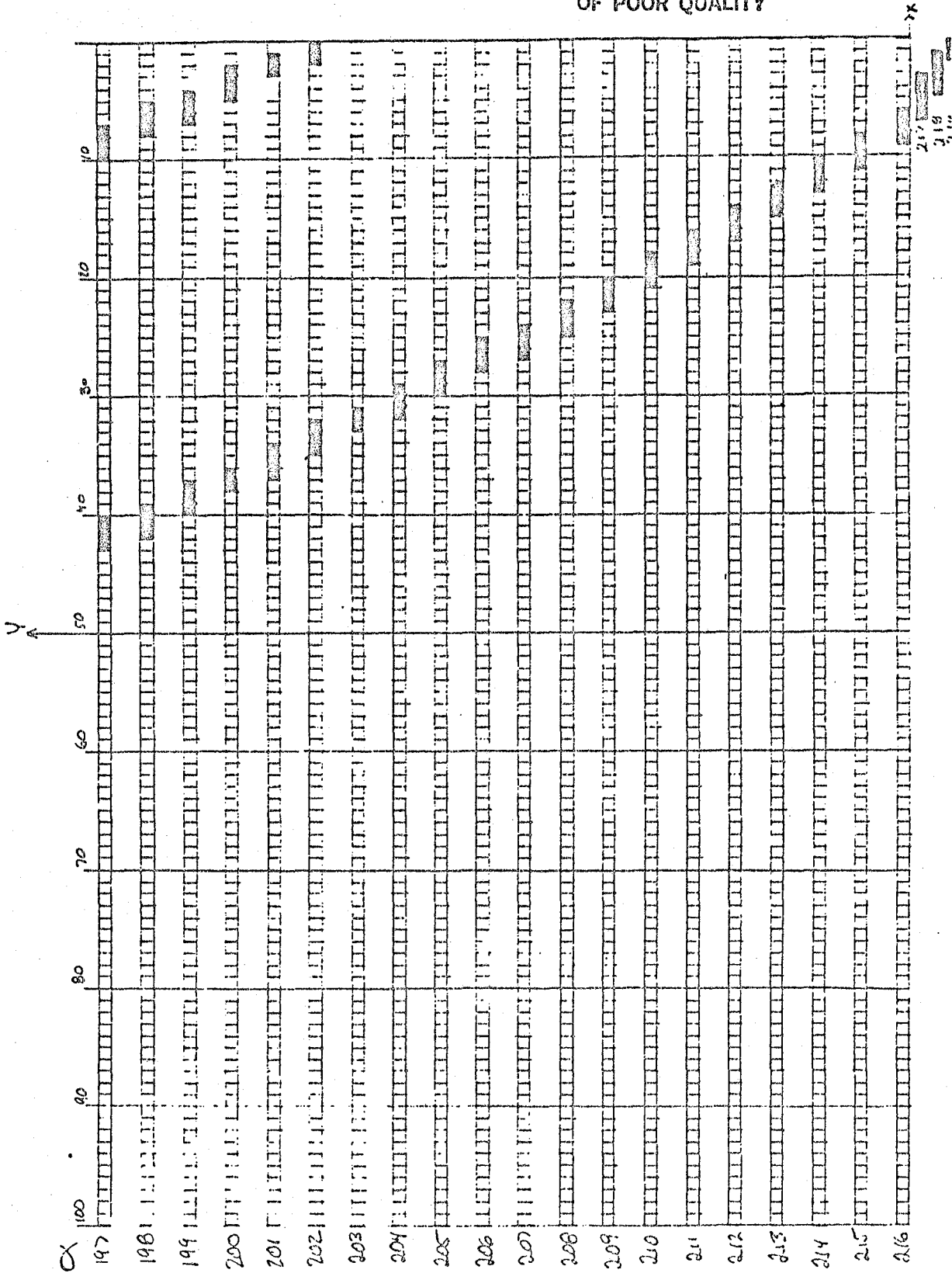
$K$   $1.7307 \text{ W/m}\cdot\text{K}$   
 $\epsilon$   $.84$   
 $G$   $.56697 \text{ W/m}^2\cdot\text{K}^4$   
 $C_p$   $1044 \text{ J/kg}\cdot\text{K}$   
 $\rho$   $1576 \text{ kg/m}^3$

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# MOVEMENT OF SHADOWS ON IRUSS MEMBER



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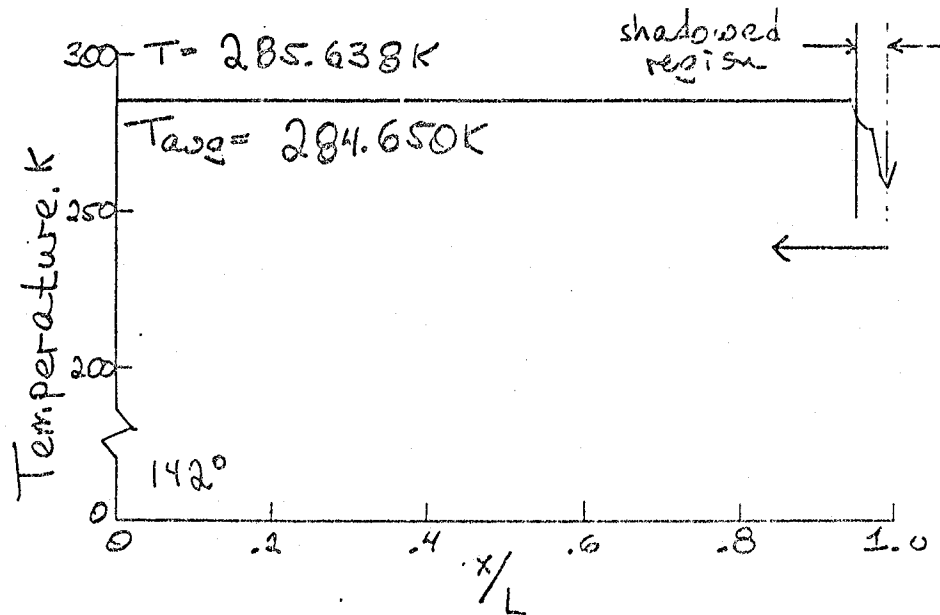
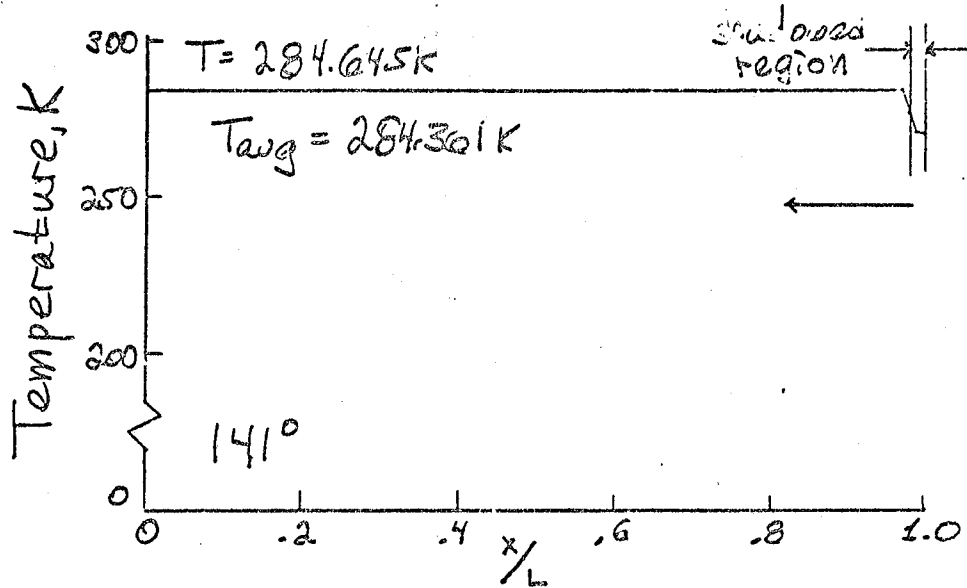


∴ x

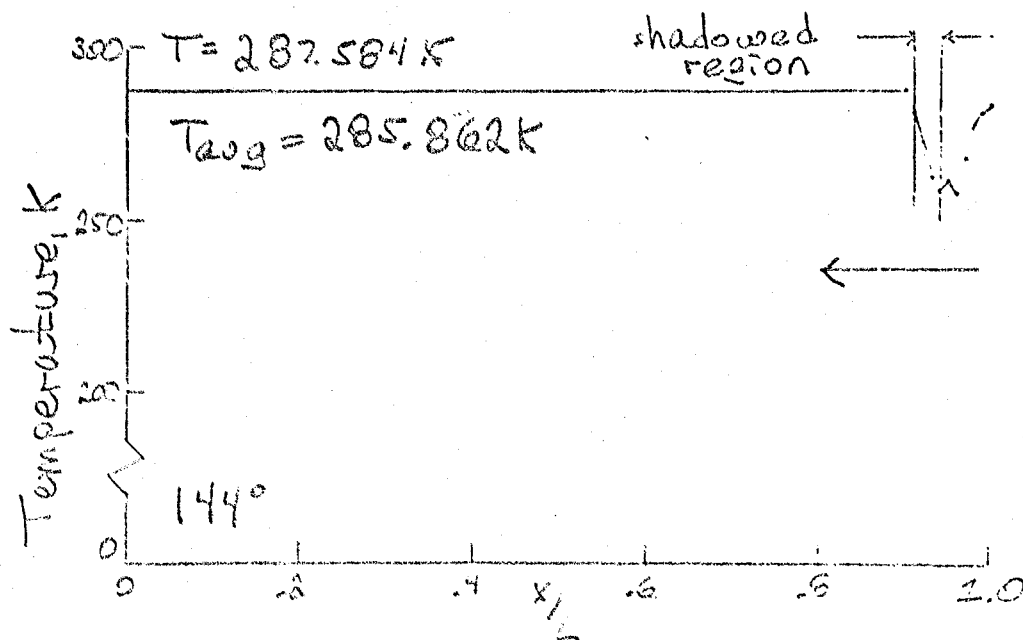
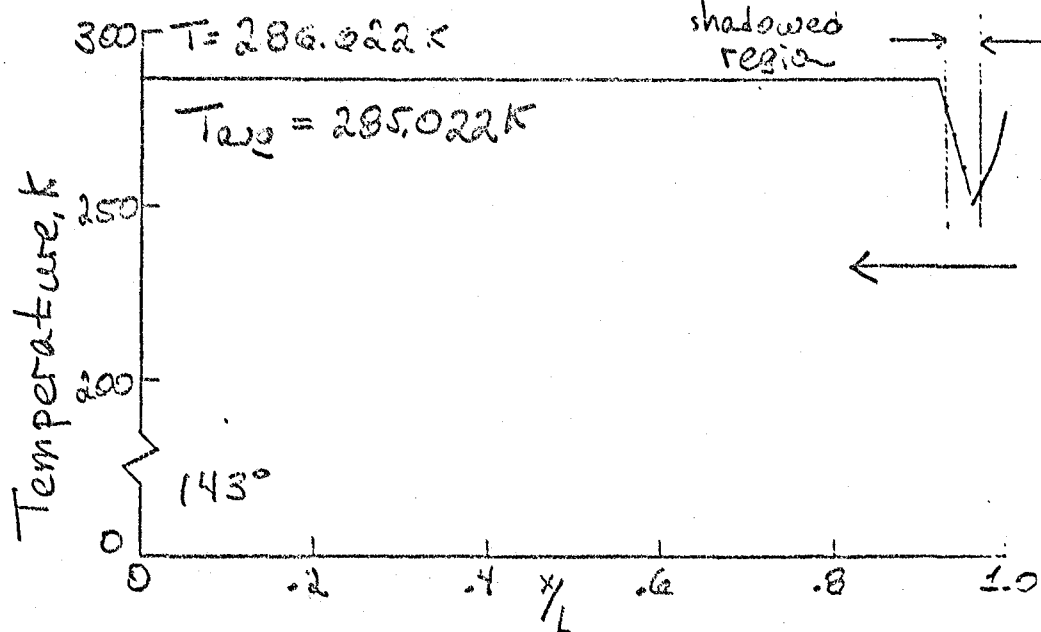
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TEMPERATURE DISTRIBUTION OF SHADOWED  
TRUSS MEMBER

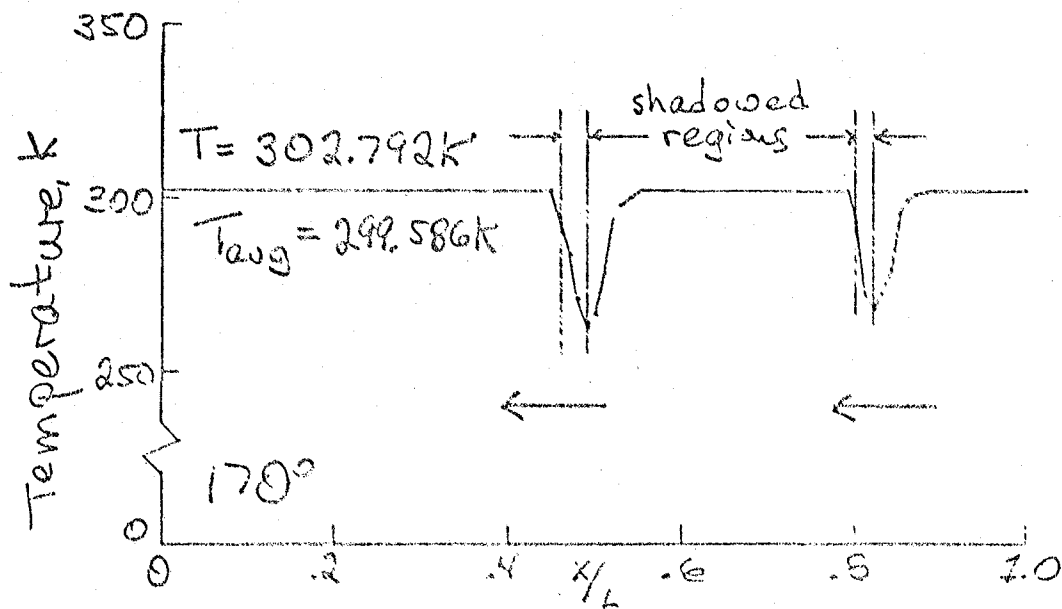
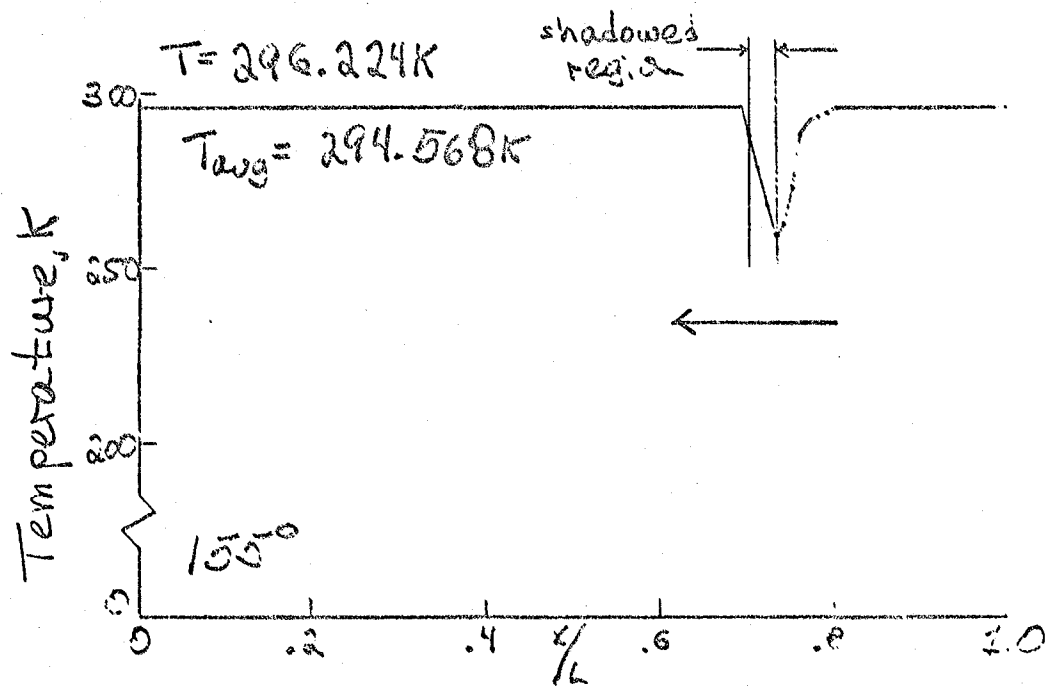


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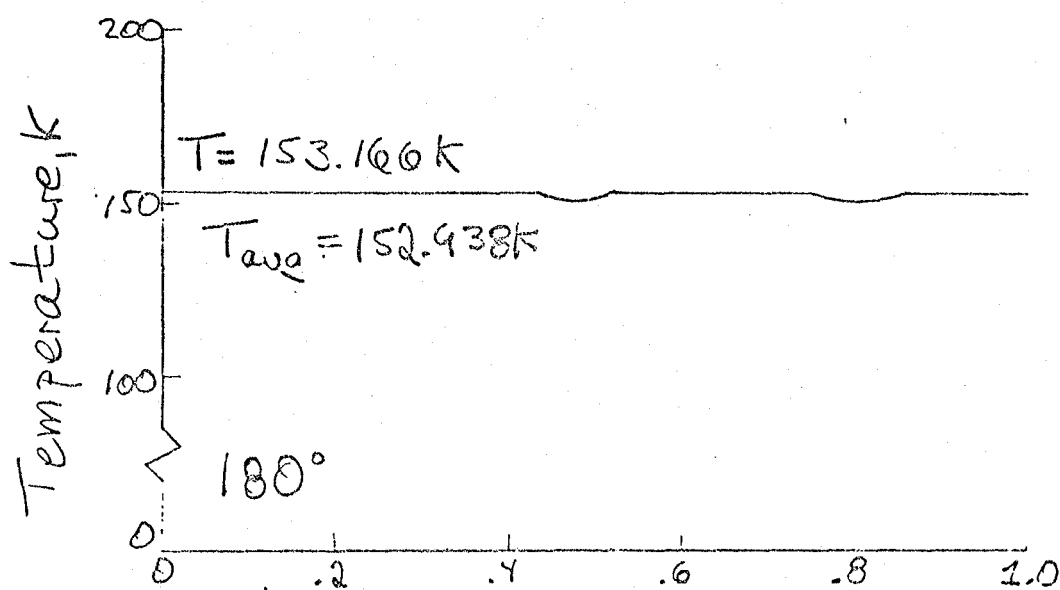
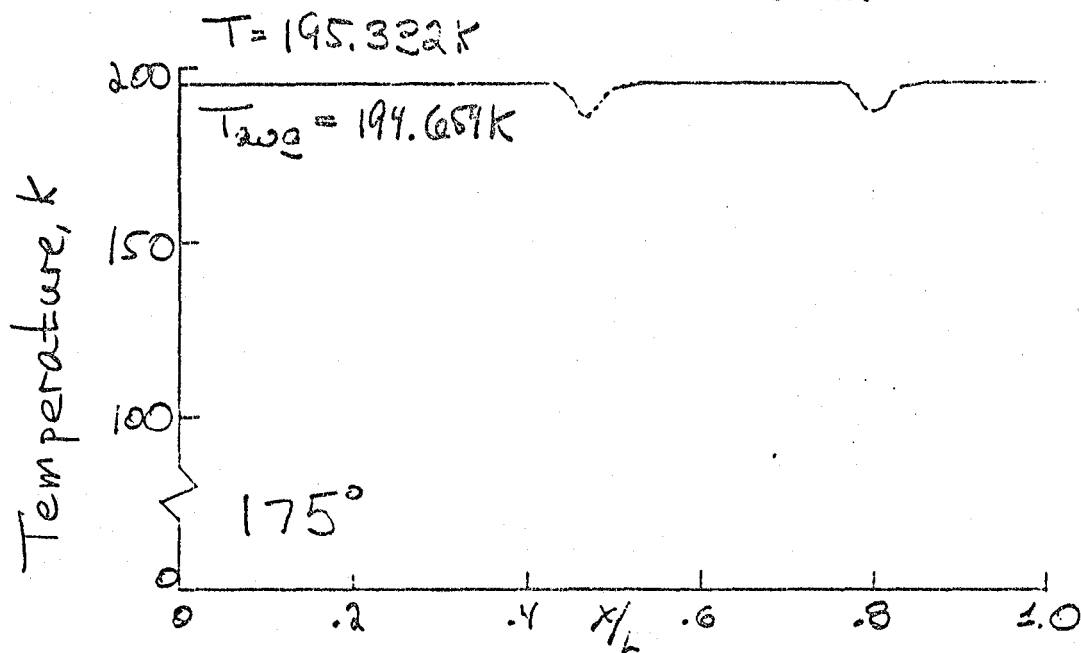




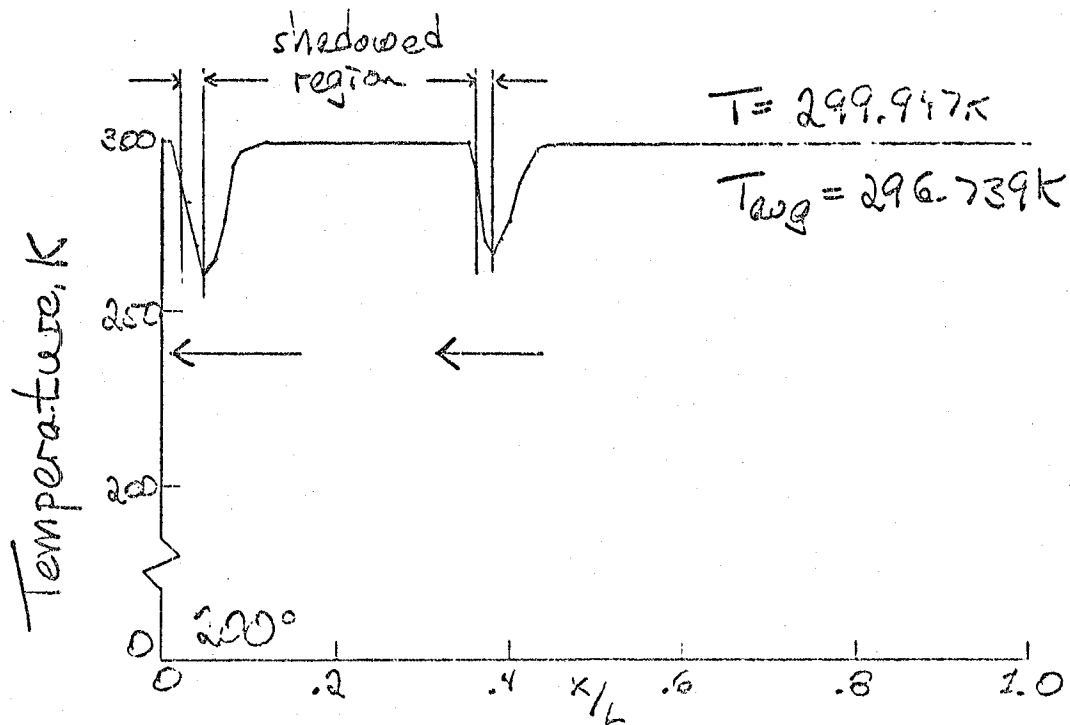
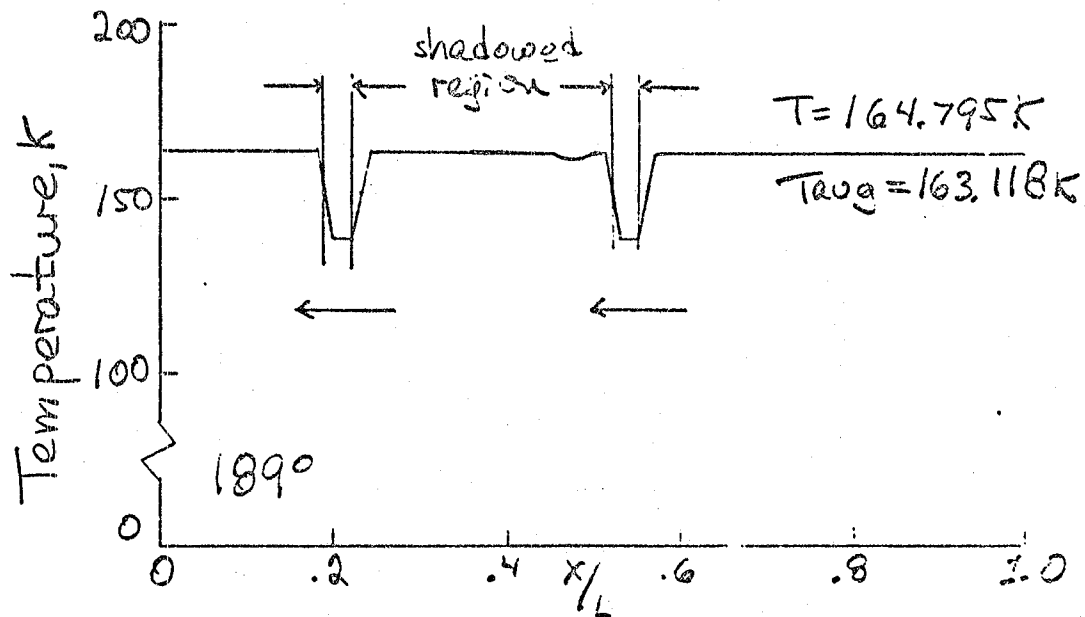
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# FINITE ELEMENT THERMAL-STRUCTURAL ANALYSIS OF CABLE SPACE STRUCTURES

- ANALYSIS INCLUDES :

- (a) CABLE PRESTRESS

- (b) APPLIED SURFACE HEATING

- (c) NONLINEAR TRANSIENT THERMAL ANALYSIS

- (d) NONLINEAR LARGE DEFORMATION STRUCTURAL ANALYSIS

- HOOP COLUMN EXAMPLE IS USED TO DEMONSTRATE THE ANALYSIS.

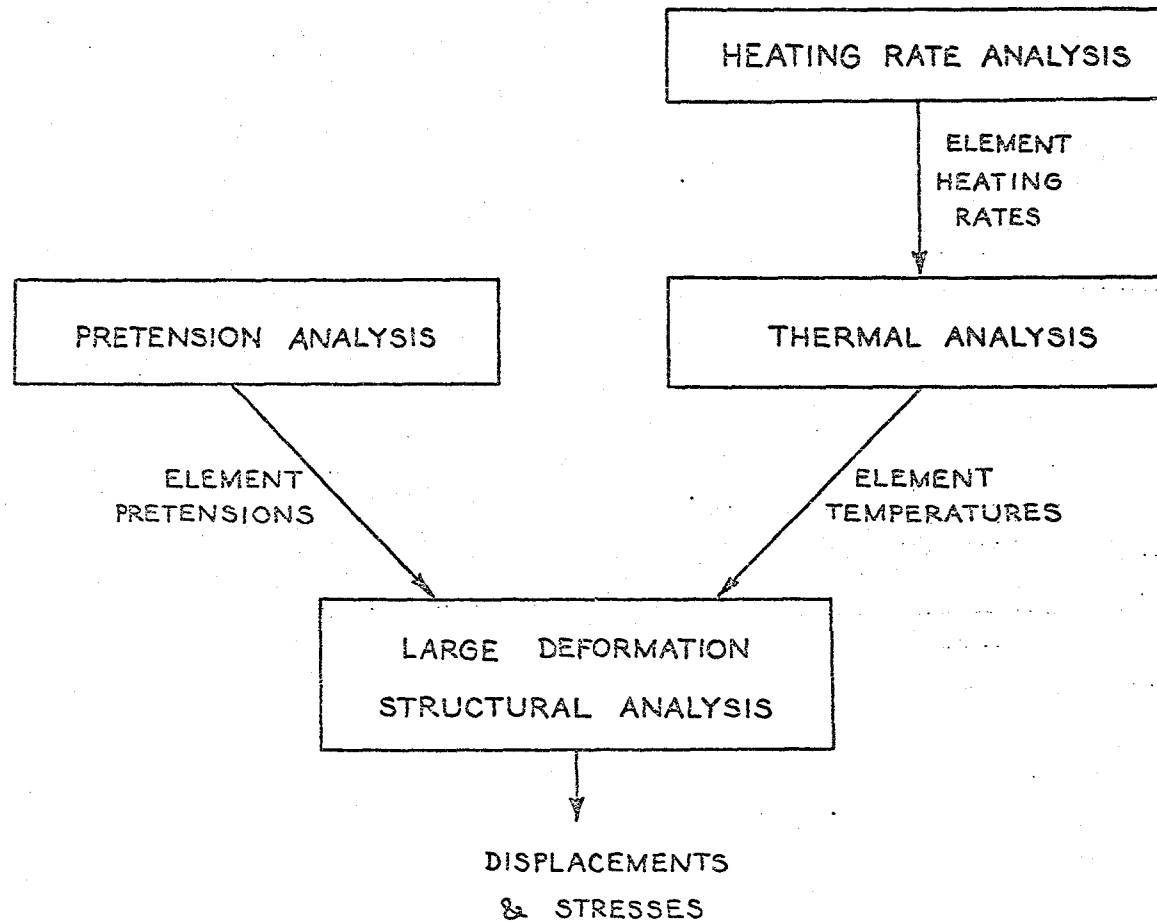
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OBJECTIVE : INVESTIGATE THREE APPROACHES FOR THERMAL-  
STRUCTURAL ANALYSIS OF CABLE STIFFENED  
SPACE STRUCTURES.

- APPROACHES :
- (1) LINEAR SMALL DISPLACEMENT STRUCTURAL  
ANALYSIS WITH PRETENSION.
  - (2) LINEAR "STRESS-STIFFENING" SMALL  
DISPLACEMENT STRUCTURAL ANALYSIS.
  - ✓ (3) NON-LINEAR LARGE DISPLACEMENT STRUCTURAL  
ANALYSIS WITH PRETENSION.

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# ANALYSIS PROCEDURES

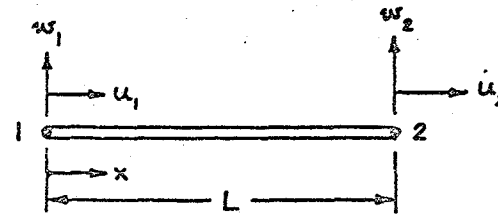


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# LARGE DEFORMATION CABLE ANALYSIS

- STRAIN - DISPLACEMENT RELATION,

$$\epsilon = \frac{\partial u}{\partial x} + \underbrace{\frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2}_{\text{NONLINEAR TERM}}$$



- FINITE ELEMENT FORMULATION;

- ESTABLISH ELASTIC STRAIN ENERGY EXPRESSION
- PERFORM MINIMIZATION WRT. NODAL DISPLACEMENTS  
TO OBTAIN F.E. EQS. IN THE FORM:

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## LARGE DEFORMATION CABLE ANALYSIS

$$\underbrace{[K_L]}_{\text{LINEAR}} + \underbrace{[K_{NL}(\delta)]}_{\text{NONLINEAR}} \{ \delta \} = \underbrace{\{ F \}}_{\text{LOADS}}$$

LOADS {

- THERMAL EFFECT
- PRETENSION EFFECT
- CONCENTRATED & DISTRIBUTED LOADS

- SOLVE THE ABOVE NONLINEAR EQS. USING ITERATIVE TECHNIQUE  
(NEWTON - RAPHSON ITERATION).

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## CABLE ELEMENTS WITH THERMAL STRAIN AND PRESTRESS

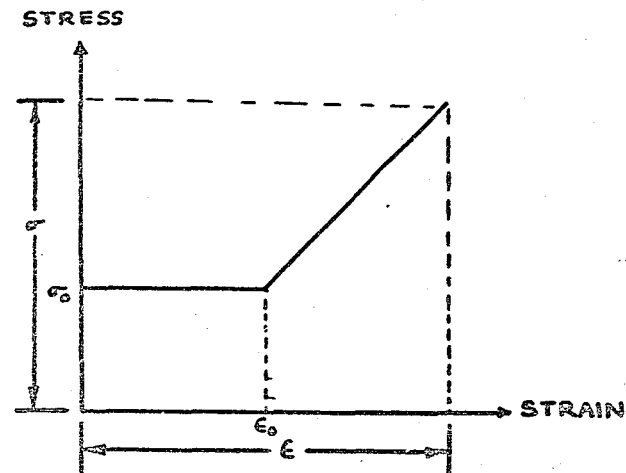
### • STRESS-STRAIN RELATION:

$$\sigma = E(\epsilon - \epsilon_0) + \sigma_0$$

WHERE

$\epsilon_0 \rightarrow$  THERMAL STRAIN

$\sigma_0 \rightarrow$  PRE-STRESS



### • DERIVATION OF F.E. EQS.;

— ESTABLISH ELASTIC STRAIN ENERGY

$$U = \frac{1}{2} \int_V (\sigma - \sigma_0)(\epsilon - \epsilon_0) dv + \int_V \sigma_0 \epsilon dv$$

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## CABLE ELEMENT WITH THERMAL STRAIN AND PRESTRESS

— EXPRESS STRAIN  $\epsilon$  IN TERMS OF DISPLACEMENTS

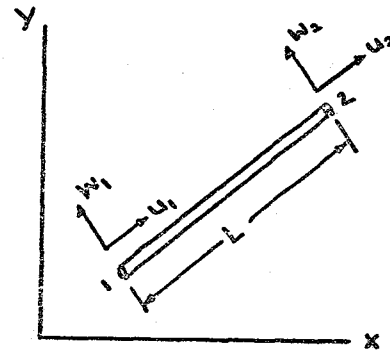
$$\epsilon = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2$$

$$\epsilon = e + \frac{1}{2} \theta^2$$

WHERE

$$e = \text{STRAIN} = \frac{u_2 - u_1}{L}$$

$$\theta = \text{ROTATION} = \frac{w_2 - w_1}{L}$$



— MINIMIZE STRAIN ENERGY W.R.T. NODAL DISPLACEMENTS  $\{u\}$

TO OBTAIN F.E. EGS.;

$$\underbrace{\begin{bmatrix} [K_L] \\ \text{LINEAR} \end{bmatrix}}_{\text{LINEAR}} + \underbrace{\begin{bmatrix} [K_{NL}] \\ \text{NONLINEAR} \end{bmatrix}}_{\text{NONLINEAR}} \{u\} = \underbrace{\begin{bmatrix} F \\ \text{THERMAL STRAIN} \end{bmatrix}}_{\text{NODAL LOADS}} \epsilon_0 - \underbrace{\begin{bmatrix} F \\ \text{PRESTRESS} \end{bmatrix}}_{\text{PRESTRESS}} \sigma_0$$

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## CABLE ELEMENT WITH THERMAL STRAIN AND PRESTRESS

### • SOLUTION METHOD

— APPLY NEWTON-RAPHSON ALGORITHM TO OBTAIN EQS.

IN THE FOLLOWING FORM

$$\left[ \bar{K}(u) \right]^m \left\{ \Delta u \right\}^{m+1} = \left\{ \bar{R} \right\}^m$$

WITH

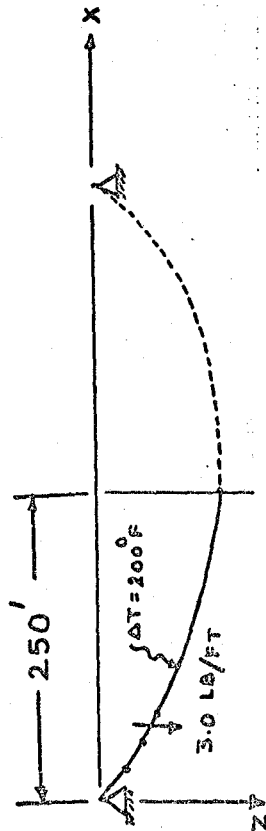
$$\left\{ u \right\}^{m+1} = \left\{ u \right\}^m + \left\{ \Delta u \right\}^{m+1}$$

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## TEST CASES

### CABLE WITH ITS OWN WEIGHT AND TEMPERATURE



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- 25 ELEMENTS ARE TAKEN AND SYMMETRY UTILIZED
- NUMBER OF ITERATION  $\approx 5$  WITH  $TOL \approx 0.001\%$
- AVERAGE ERROR  $\approx 0.02\%$  IN DISPLACEMENTS

## EXACT SOLUTION OF CATENARY PROBLEM

DEFLECTION DUE TO OWN WEIGHT:

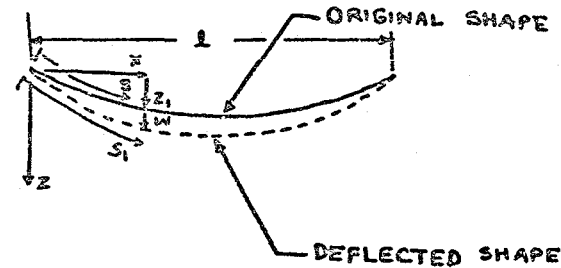
"INEXTENSIBLE"

-  $x, z$  AND  $s$  FOR ORIGINAL SHAPE ARE

$$z = z_1 = \frac{H}{m_g} \left[ \cosh \left( \frac{m_g l}{2H} \right) - \cosh \frac{m_g}{H} \left( \frac{l}{2} - x \right) \right]$$

$$s = \frac{H}{m_g} \left[ \sinh \left( \frac{m_g l}{2H} \right) - \sinh \frac{m_g}{H} \left( \frac{l}{2} - x \right) \right]$$

$$H \text{ IS SOLVED FROM } \sin \left( \frac{m_g l}{2H} \right) = \frac{m_g L_0}{2H}$$



"EXTENSIBLE"

- FOR DEFLECTED SHAPE,  $H$  IS SOLVED FROM EQS.

$$\sin \left( \frac{Wl}{2HL_0} - \frac{W}{2EA_0} \right) = \frac{W}{2H}$$

CO-ORDINATES OF DEFLECTED POINTS ARE GIVEN BY

$$x = \frac{HS}{EA_0} + \frac{HL_0}{W} \left[ \sin^{-1} \left( \frac{W}{2H} \right) - \sin^{-1} \left\{ \frac{W \left( \frac{1}{2} - \frac{s}{L_0} \right)}{H} \right\} \right]$$

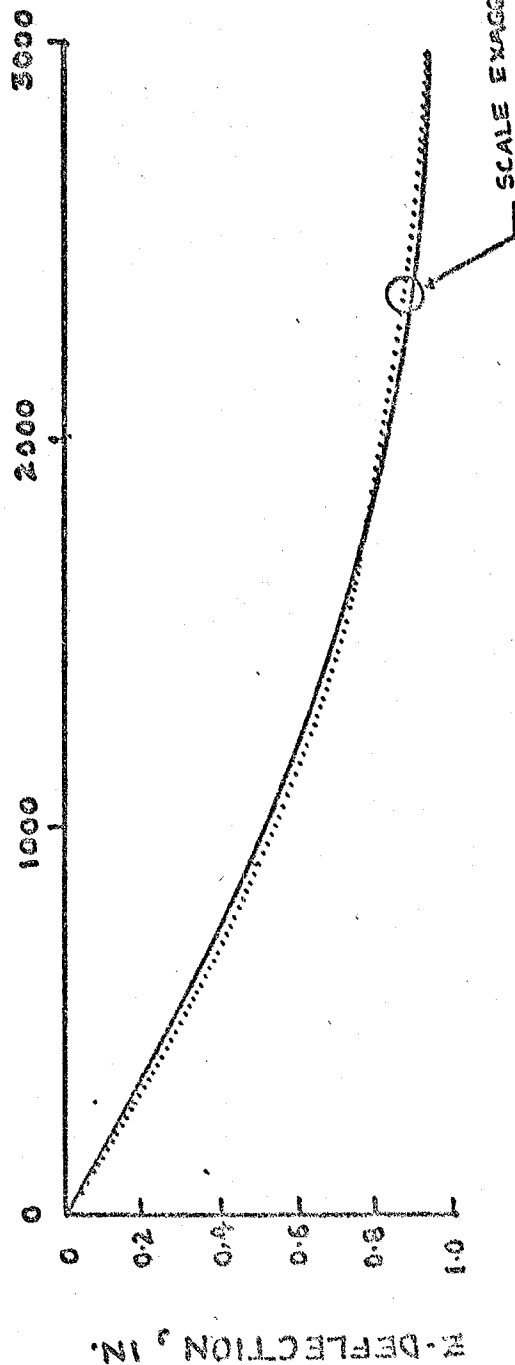
$$z = z_2 = \frac{WS}{EA_0} \left( \frac{1}{2} - \frac{s}{L_0} \right) + \frac{HL_0}{W} \left[ \left[ 1 + \left( \frac{W}{2H} \right)^2 \right]^{1/2} - \left[ 1 + \frac{W^2}{H^2} \left( \frac{1}{2} - \frac{s}{L_0} \right)^2 \right]^{1/2} \right]$$

DEFLECTION IN  $z$ -DIRECTION IS  $w = z_2 - z_1$

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# DEFLECTION OF CATENARY DUE TO WEIGHT

SPAN, INCHES

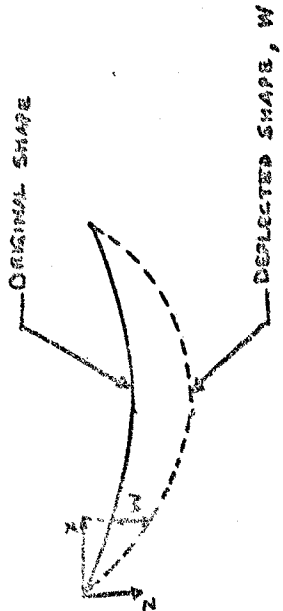


— EXACT SOLUTION

..... F.E. SOLUTION

# EXACT SOLUTION OF CATENARY PROBLEM

DEFLECTION DUE TO OWN WEIGHT AND TEMPERATURE RISE:



$$W = \left( \frac{mg\lambda^2}{2H} \right) \left[ \frac{\bar{h}}{(1-\bar{h})} \frac{1}{\lambda} \left( 1 - \frac{x}{\bar{h}} \right) \right]$$

$\bar{h}$  IS ROOT (POSITIVE) OF EGS.

$$\bar{h}^3 - (2 + \theta + \frac{\lambda^2}{24}) \bar{h}^2 + (1 + 2\theta + \frac{\lambda^2}{12}) \bar{h} - \theta = 0$$

WHERE

$$\theta = \alpha |\Delta T| L_0 / (H L_0 / EA)$$

$$L_0 = L \left[ 1 + \left( \frac{mg\lambda^2}{H} \right) / 12 \right]$$

$$L = L_0 \left[ 1 + e \left( \frac{\lambda}{\bar{h}} \right)^2 \right]$$

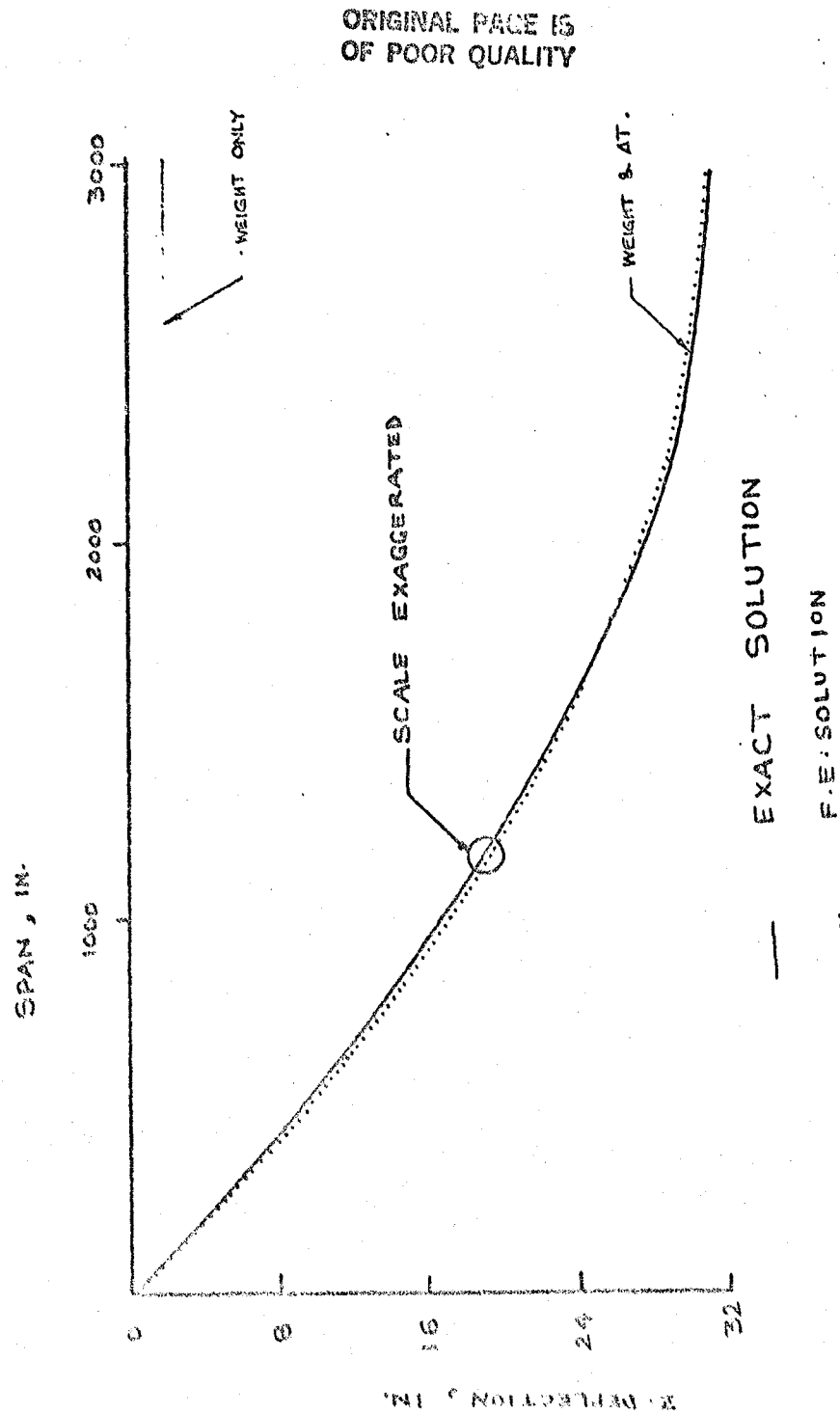
$$\lambda^2 = \left( \frac{mg\lambda^2}{H} \right)^2 L / (H L_0 / EA)$$

$\alpha$  = COEFF. OF THERMAL EXPANSION

$\Delta T$  = RISE IN TEMPERATURE.

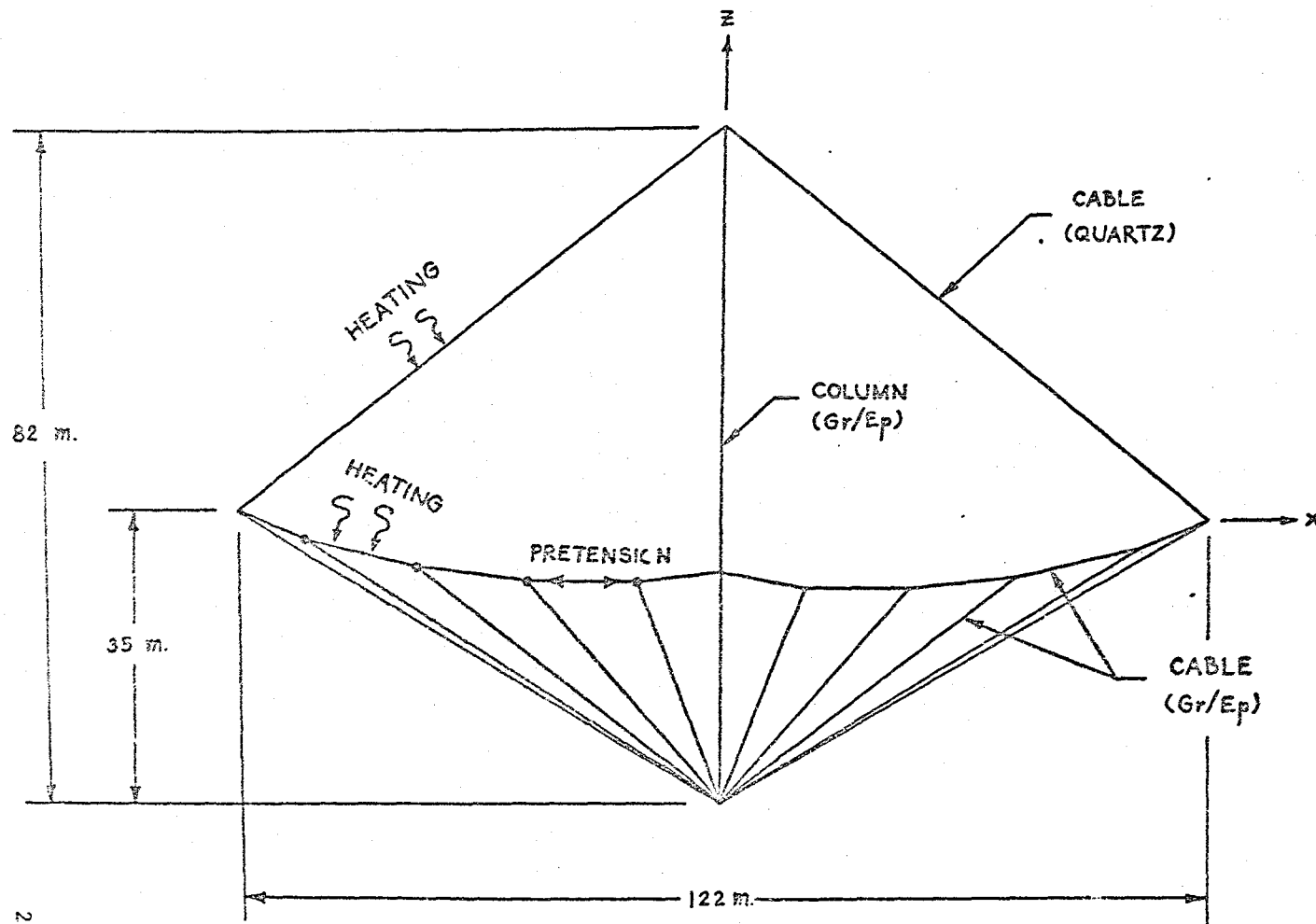
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# DEFLECTION OF CATENARY DUE TO WEIGHT AND TEMPERATURE

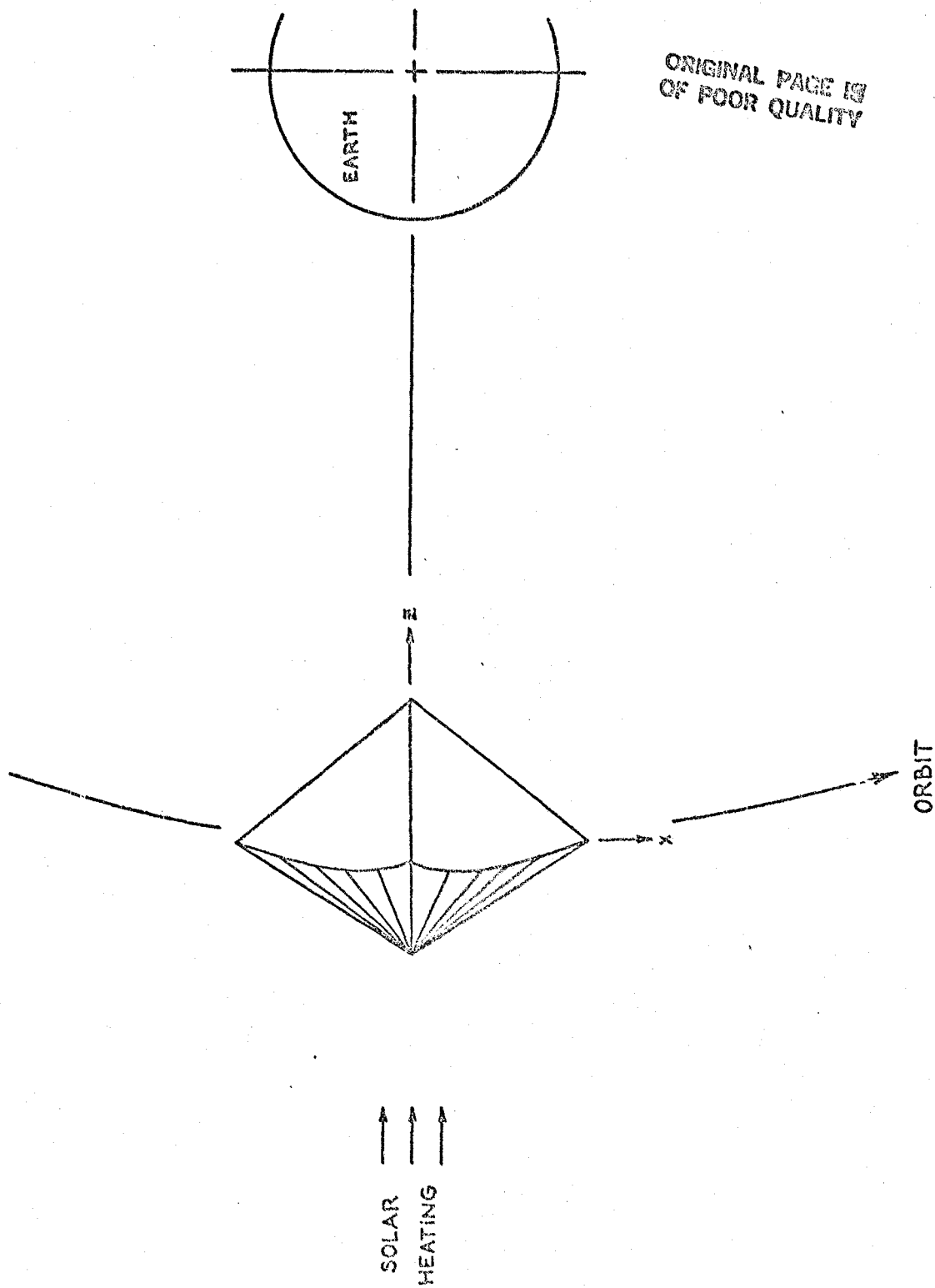




# THERMAL-STRUCTURAL ANALYSIS OF HOOP COLUMN (IDEALIZED 2-D MODEL)

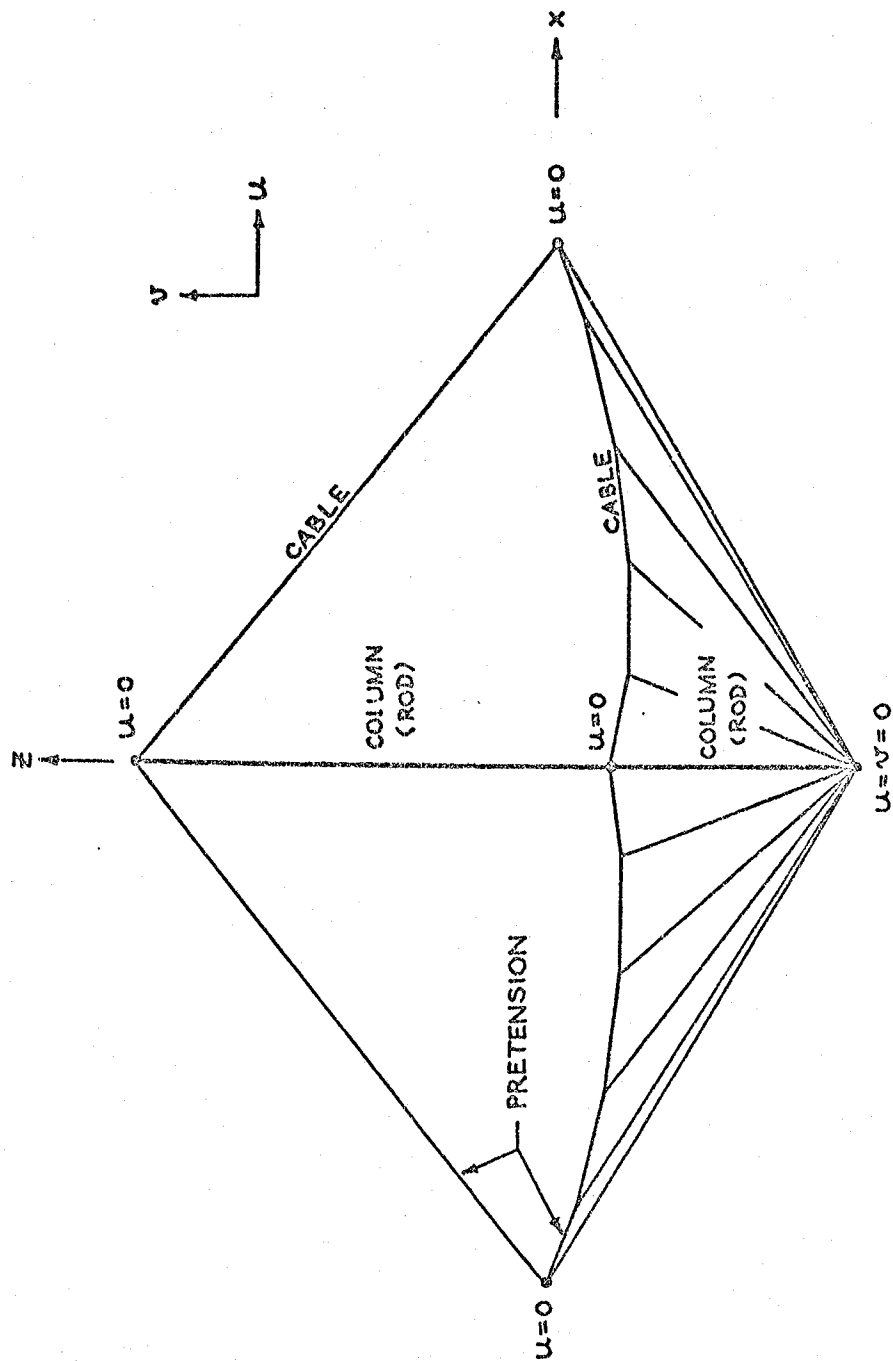


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# HOOP COLUMN BOUNDARY CONDITIONS

## (IDEALIZED 2-D MODEL)

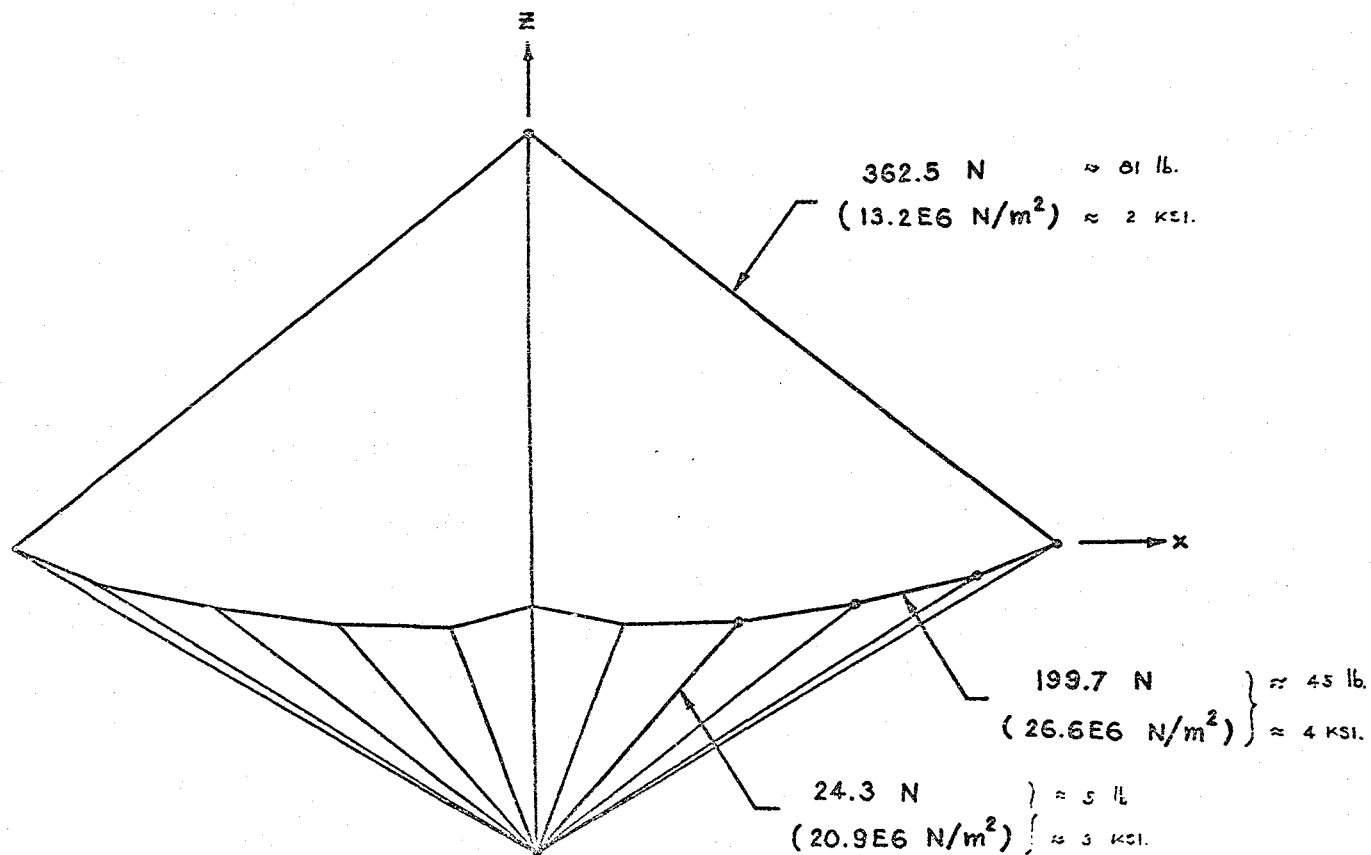


## PRETENSION ANALYSIS

- USING A COMPUTER CODE DEVELOPED, CABLE PRETENSIONS ARE COMPUTED SUCH THAT THE STRUCTURE IS IN:
  - REQUIRED GEOMETRY
  - EQUILIBRIUM
- CABLE PRETENSIONS OBTAINED ARE TRANSFERRED DIRECTLY TO THE LARGE DEFORMATION STRUCTURAL ANALYSIS PROGRAM (AS PRETENSION EFFECT) WHICH WILL BE INCLUDED IN THE FINAL DISPLACEMENT AND STRESS COMPUTATIONS.

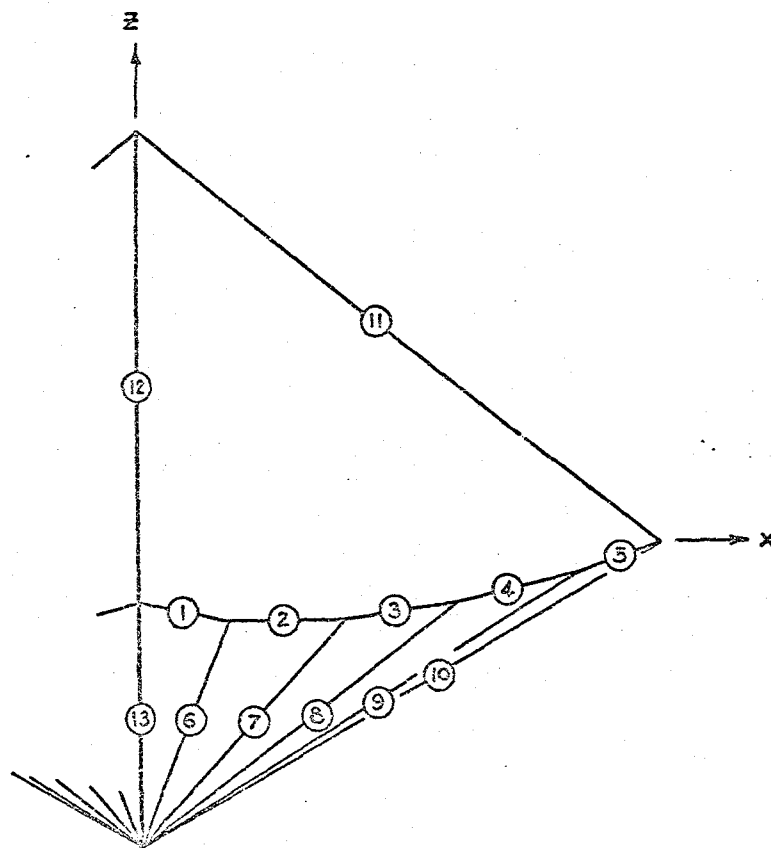
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# TYPICAL RESULT OF CABLE PRETENSIONS



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# RESULT OF CABLE PRETENSIONS



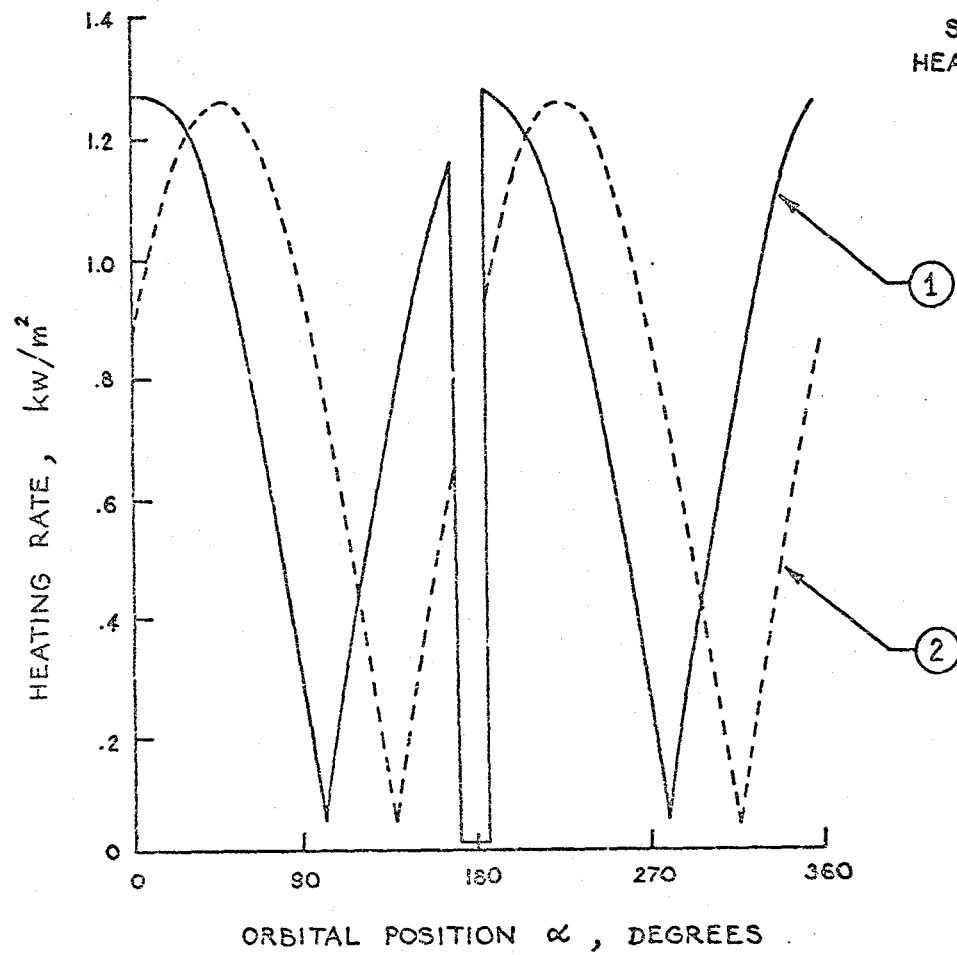
ELEMENT No.	FORCE (N)	STRESS (N/m <sup>2</sup> )
1	141	19.E6
2	152	20.E6
3	170	23.E6
4	200	27.E6
5	452	60.E6
6	43	37.E6
7	24	21.E6
8	33	21.E6
9	258	222.E6
10	140	140.E6
11	363	13.E6
12	-511	-2.E6
13	-580	-2.E6

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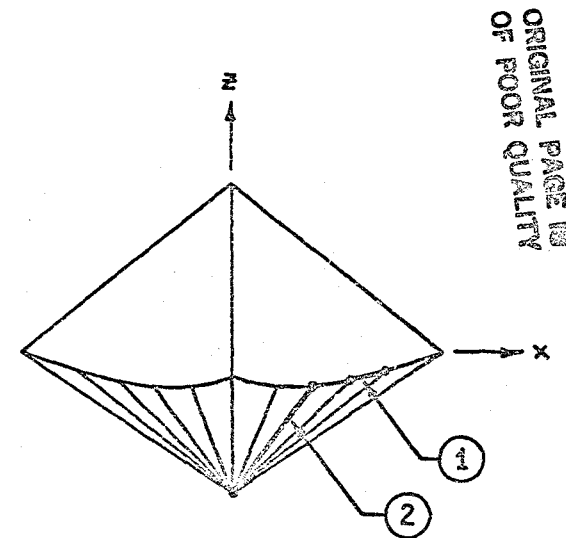
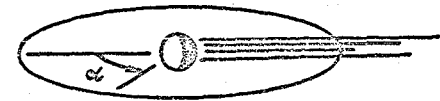
## HEATING RATE ANALYSIS

- HEAT SOURCES INCLUDE ;
  - SOLAR
  - EARTH EMISSION
  - EARTH ALBEDO
- ELEMENT HEATING RATES ARE COMPUTED AT DIFFERENT ORBITAL POSITIONS FOR AN ORBIT.
- THESE ELEMENT HEATING RATES ARE TRANSFERRED DIRECTLY TO THE THERMAL ANALYSIS PROGRAM FOR COMPUTATION OF ELEMENT TEMPERATURE RESPONSES.

# TYPICAL CABLE SURFACE HEATING RATES



SOLAR HEATING →





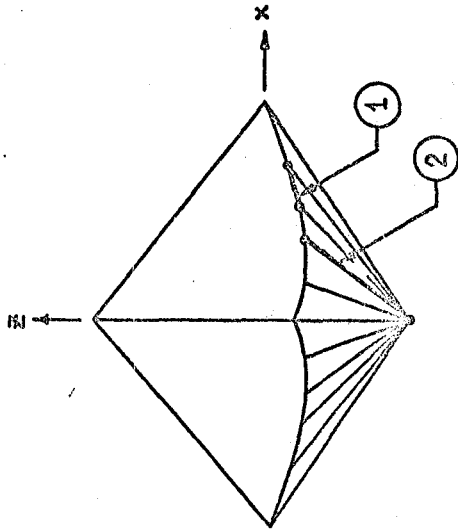
## THERMAL ANALYSIS

- ASSUME ISOTHERMAL ELEMENT TEMPERATURES.
- ANALYSIS (NONLINEAR TRANSIENT) INCLUDES:
  - APPLIED SURFACE HEATING
  - SURFACE RADIATION TO SPACE
- ELEMENT TEMPERATURES ARE COMPUTED AT DIFFERENT ORBITAL POSITIONS FOR AN ORBIT.
- ELEMENT TEMPERATURES ARE TRANSFERRED TO THE LARGE DEFORMATION STRUCTURAL ANALYSIS PROGRAM (AS THERMAL EFFECT) FOR DISPLACEMENT AND STRESS COMPUTATIONS.

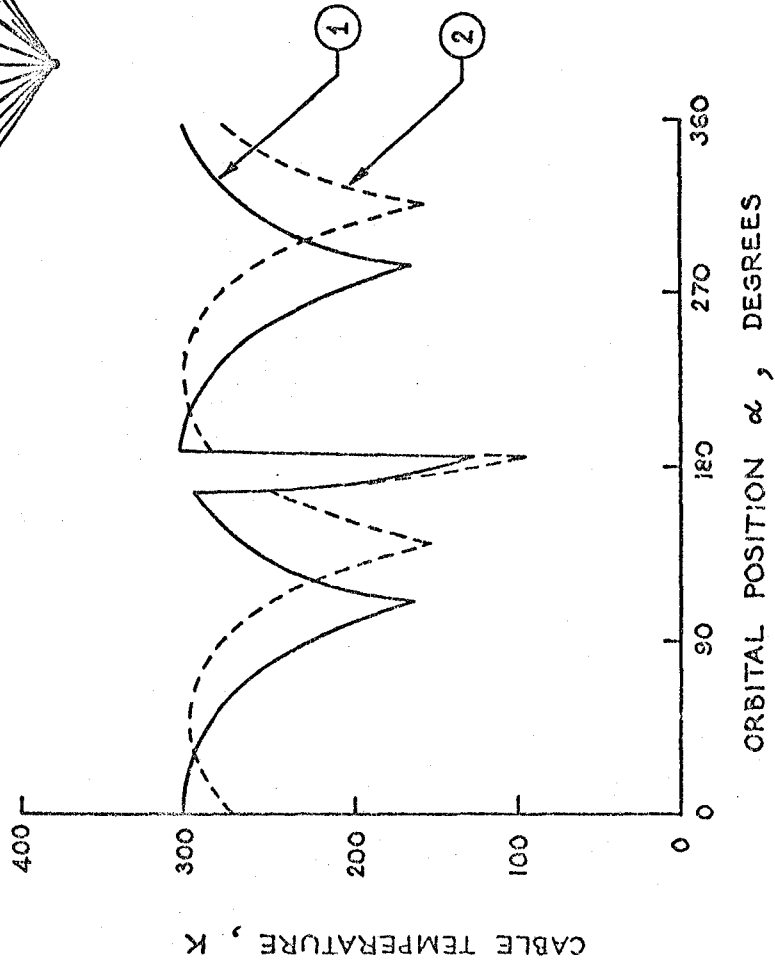
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# TYPICAL CABLE TEMPERATURE RESPONSE

SOLAR  
HEATING



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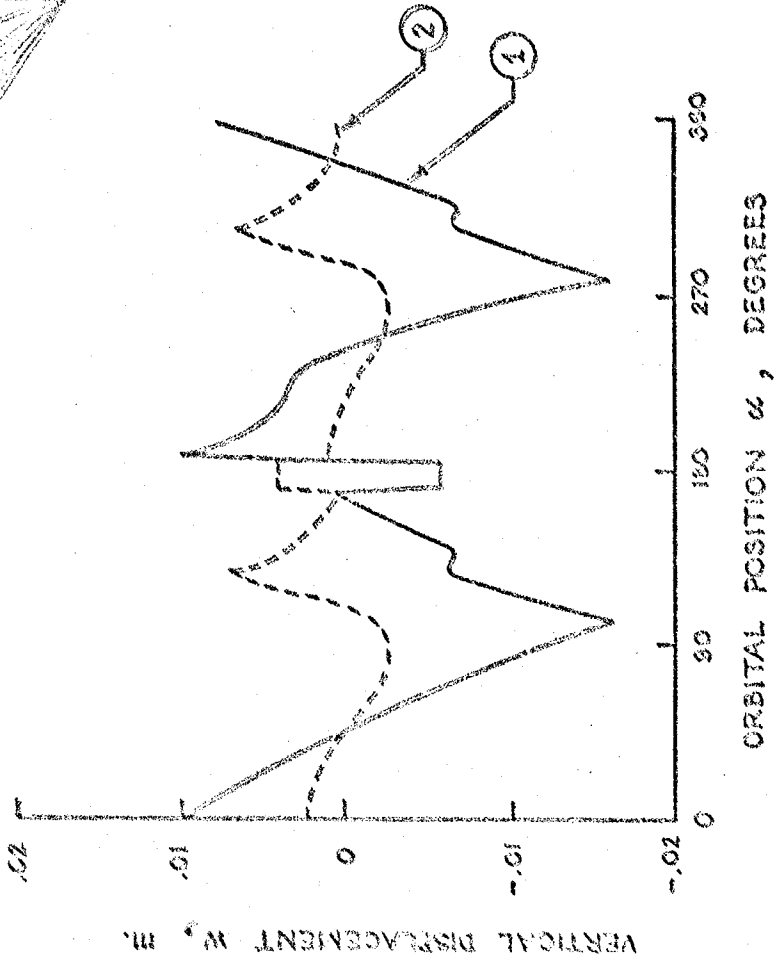
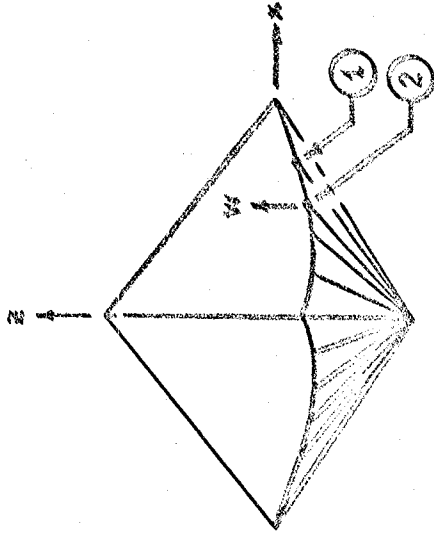
## STRUCTURAL ANALYSIS

- ANALYSIS INCLUDES :
  - LARGE DEFORMATION (NONLINEAR)
  - THERMAL EFFECT
  - PRETENSION EFFECT
- DISPLACEMENTS AND STRESSES ARE COMPUTED AT DIFFERENT ORBITAL POSITIONS FOR AN ORBIT (QUASI-STATIC)



# TYPICAL NODAL DISPLACEMENT HISTORIES

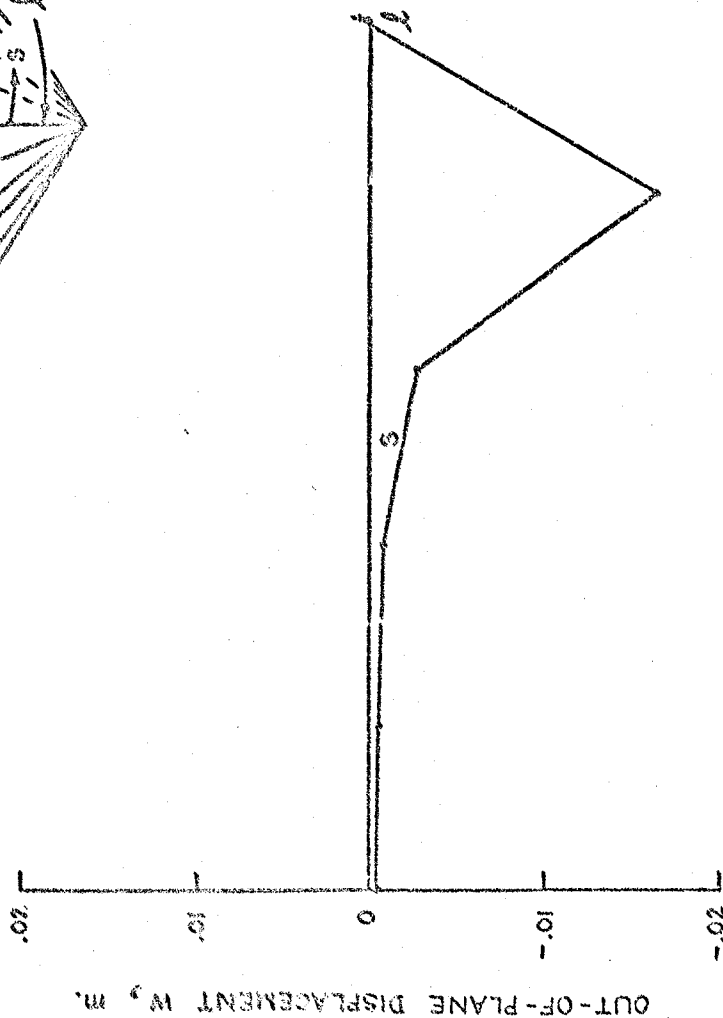
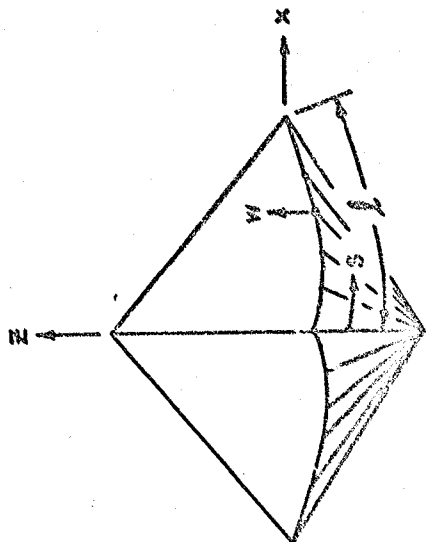
SOLAR  
HEATING



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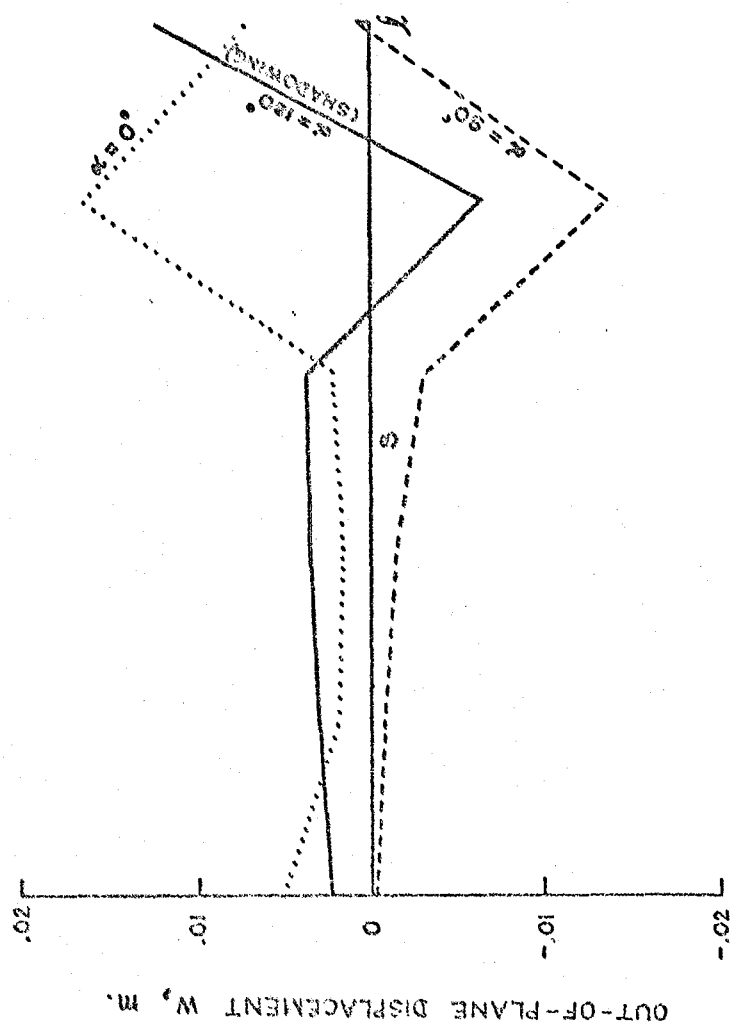
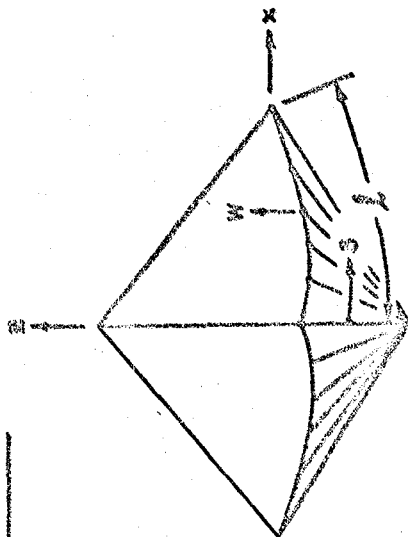
# OUT-OF-PLANE DISPLACEMENT DISTRIBUTION AT $\alpha = 100^\circ$

SOLAR  
HEATING



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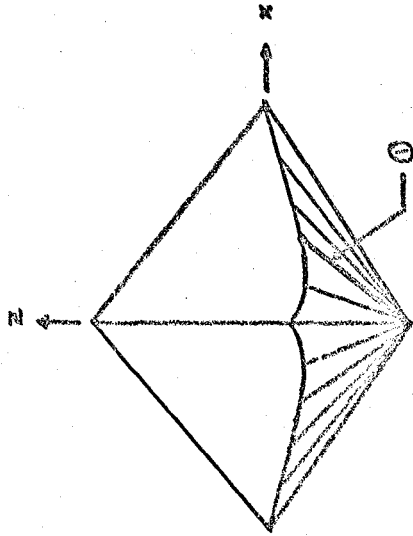
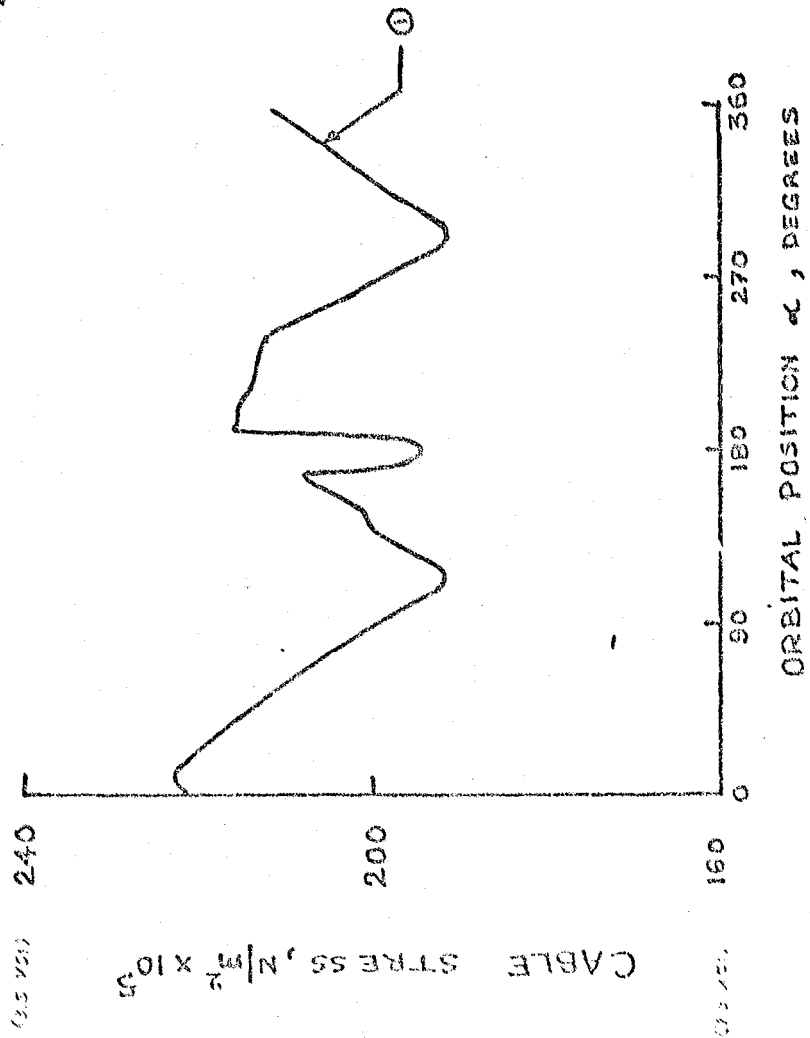
# OUT-OF-PLANE DISPLACEMENT DISTRIBUTIONS AT DIFFERENT ORBITAL POSITIONS



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# TYPICAL CABLE STRESS RESPONSE

SOLAR  
HEATING



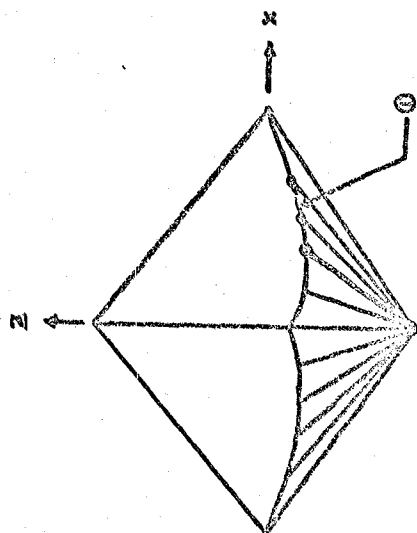
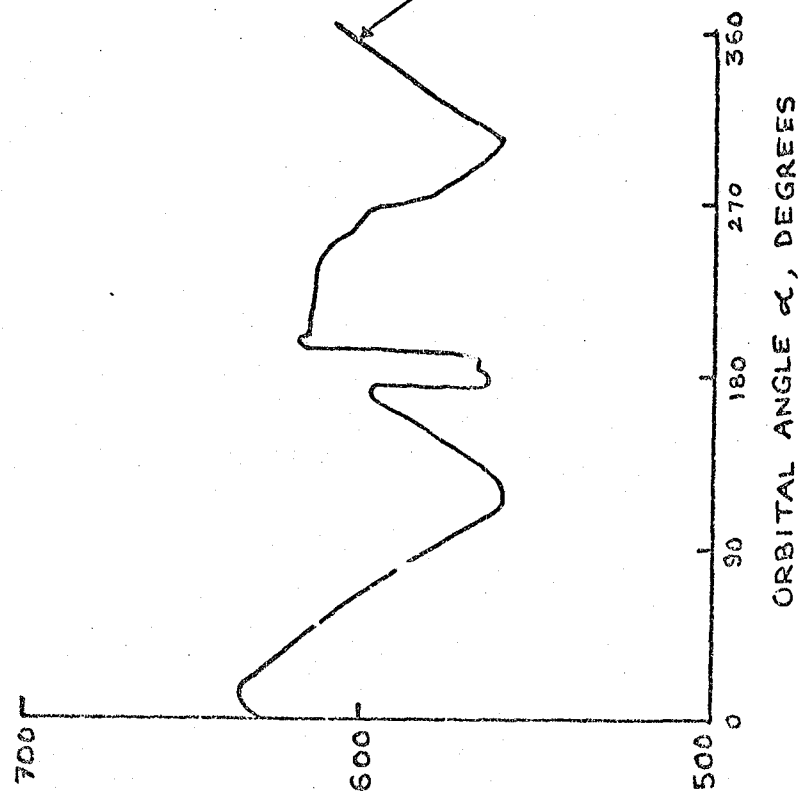
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# TYPICAL CABLE STRESS RESPONSE



SOLAR  
HEATING

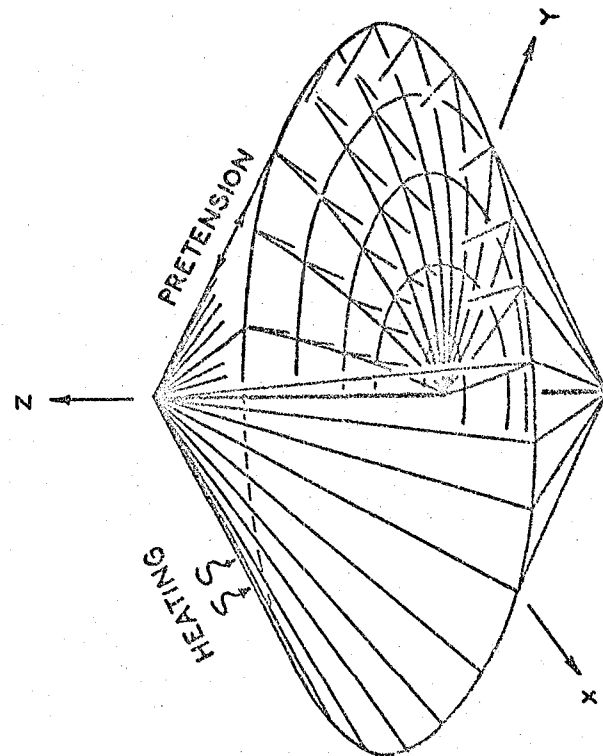
CABLE STRESS,  $N/m^2 \times 10^5$



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# 3-D HOOP COLUMN MODEL



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( 123 NODES, 386 ELEMENTS )

## PLANS :

- COMPLETE DEVELOPMENT OF PRETENSION PROGRAM.
- PERFORM 2-D HOOP COLUMN MODEL WITH :
  - (1) LINEAR ANALYSIS
  - (2) LINEAR "STRESS-STIFFENING" ANALYSIS (ANSYS)
- PERFORM 3-D HOOP COLUMN MODEL WITH :
  - (1) LINEAR ANALYSIS
  - (2) LINEAR "STRESS-STIFFENING" ANALYSIS
  - (3) NON-LINEAR ANALYSIS
- MEMBRANE EFFECTS (?)
  - THERMAL ANALYSIS
  - STRUCTURAL ANALYSIS

**End of Document**